Confirmatory Factor Analysis Approach: A Case Study of Mathematics Students' Achievement in TIMSS

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ABSTRACT

Mathematics is king and an important subject taught in school. However, student achievement in this subject is declining. The aim of this study is to identify the factors that affecting student's achievement in mathematics using confirmatory factor analysis and to investigate the relationship between factors of the student's achievement in mathematics. In this study, we de developed a model and test whether the data fit with hypothesized model. The data were obtained from Trend in International Mathematics and Science Study (TIMSS). The study of confirmatory factor analysis on student achievement in mathematics via 4 indicators which are: 1) motivation 2) teacher's role 3) attitude 4) self-confidence. The model developed of confirmatory factor analysis on the student's achievement in mathematics fit well with the empirical data set.

Keywords: Confirmatory factor analysis, model fit, students' achievement, TIMSS

1. Introduction

Malaysia needs more experts in the field of science and technology. Science and technology require them to be able to assimilate and evaluate information in mathematics. It is shows that mathematics is important and need to be mastered by students. Mathematics proficiency is important due to this subject is a king of science. Mathematics is a compulsory subject taught in every school. However, Malaysian student's scores in global tests

of mathematics is poor. This issue often get public and government attention since the results of students in math achievement are in declining at the international level. TIMSS results show that Malaysian student achievement in mathematics has been declining from 16 in 1999 to 26 in 2011 (NCES, 2014). The government also has planned a strategy and give a lot of provision to improve the student's achievement. However, the students achievement in mathematics still decline. Therefore, a comprehensive approach needs to be done in improving the achievement in mathematics. Thus, mathematical model such as confirmatory factor analysis is an alternative approaches to study which factors affecting the achievement of students in mathematics. Confirmatory factor analysis or CFA is a statistical modelling technique used to estimate the parameter of the models and it is mostly used in psychology research. CFA model include the latent variables and observed variables. Usually, CFA is used to measure the observed variables and latent variables which is specified by the theoretical construct. Theoretical construct usually is based on the previous study. CFA is tools to test the unidimensionality, reliability and validity of the models.

No doubt there are many studies on student achievement has been done in Malaysia. Hassan and Ab. Aziz (2011) study the level of the interest in mathematics within the students, the factors affecting in student interest in mathematics and the problems of the lack interest students in mathematics. Overall, the results of the study shows the interest of the study is a major factors affecting the student's mathematics achievement. The major problems of the students which have a lack of interest in mathematics is due to their selves, teachers, parents, friends and environment.

Aziz, et al (2006) in his study tried to determine the relationship between motivations, learning style with additional mathematic achievement of students in form 4. He also identify the different learning style based on gender, ethnic and its relationship with early results in mathematics. Descriptive analysis shows that the students who achieved excellent results in mathematics have high internal motivation compare to external motivation. This research revealed that motivation plays an important role in determining student choice of learning style.

Therefore, the proposed of this study have two fold: first, is to identify the factors affecting student's achievement in mathematics using confirmatory factor analysis. Second is to investigate the relationship between factors of the student's achievement in mathematics

2. Data and Methodology

In this research, we used the secondary of Trend in International Mathematics and Science Study (TIMSS) year 2011data and it obtained from NCES (2014). The sample group consist of 2500 grade 8 students in 92 schools from Malaysia. All participants were given the questionnaire to answer. The questionnaire include background and curriculum questions. Students were also given the assessment to measure their understanding in mathematics. The approach of confirmatory factor analysis is to verify the factors that contribute to student's achievement in math.

Factor Analysis

Factor analysis is a statistical method which is very famous in the research involving a questionnaire. There are two type of factor analysis which is exploratory factor analysis (EFA) and confirmatory factor analysis (CFA). Factor analysis approach is commonly found in psychology, education and health related professions The study of the area is usually used a questionnaire as a method of study and factor analysis is use to interpret the data appropriately. Factor analysis is a method which also have many other uses. Besides reduces number of factors, it also provide construct validity evidence of self-reporting scales (Nunnally, 1978).

Confirmatory factor analysis

CFA is run in the measurement model. There are two ways to run the CFA which are by individual confirmatory factor analysis or pool confirmatory factor analysis. It's depends on how many items in the constructs. For example, If the item in construct have more than four, it should us individual factor analysis, and run the measurement model separately or one by one. While, pooled CFA run all the measurement model at the same time. Pooled CFA could be used if the factors have less than four item in each construct (Joreskog, 1978).

The parameter of the models include factor loadings, variances, covariance and residual error variance of the observed variables are estimated using CFA. Besides that, CFA also measure the goodness-of-fit in the model (Bentler and Douglas, 1980). Several measures such as absolute fit, incremental fit and parsimonious fit with the basic indication are used to check how well the proposed model fit the data.

For example, the value in absolute fit indices shows how strong the model fits with the data and proposed the model which has the best fit (McDonald and Ho, 2002). The indices used to assess the measurement models fit to the data (Yuan, 2005) are shown in Table 1.

TABLE 1**:** Index category and the level of acceptances

Name of category	Name of index	Index full name	Level acceptance
Absolute fit	Chisq	Discrepancy Chi Square	P > 0.05
	RMSEA	Root Mean Square of Error Approximation	RMSEA > 0.08
	GFI	Goodness of Fit Index	GFI > 0.90
Incremental fit	AGFI	Adjusted Goodness of Fit	AGFI > 0.90
	CFI	Comparative Fit Index	CFI > 0.90
	TLI	Tucker-Lewis Index	TLI > 0.90
	NFI	Normed Fit Index	NFI > 0.90
Persimonious fit	Chisq/df	Chi Square/Degree of Freedom	Chi square / df < 5.0

The formula to compute the average variance extracted and composite reliability is shown in Table 2. These two value are used to determine the validity and reliability of the model.

TABLE 2: Formula for Average Variance Extracted (AVE) and Composite Reliability (CR)

$$AVE = \sum K^2/n$$

K = factor loading of every item
n = number of items in a model

$$CR = (\sum K)^2/[(\sum K)^2 + (\sum 1 - K^2)]$$

3. Result and Discussion

Unidimensionality

Unidimensionality is a test for identify every item in factors achieved their minimum requirement for acceptable factor loadings (Awang, 2012). Thus, the procedure is to delete the lowest factor loading the items in latent construct one by one until its values over 0.5. After that, the measurement model should be test again until the unidimensionality requirement achieved. Table 3 show the results of factor loading for every item. From

that table show that all the factor loadings value are above 0.5. Means that all the items achieved the unidimensionality.

TABLE 3: Factor loading of the items

Construct	Item	Factor Loading
Motivation	F1a	0.766
	F1b	0.714
	F1c	0.714
	F1d	0.659
	F1e	0.608
Teacher's Role	F2a	0.655
	F2b	0.728
	F2c	0.555
	F2e	0.766
Attitude	F3a	0.804
	F3b	0.510
	F3c	0.715
Self-Confidence	F4a	0.580
	F4b	0.650
	F4c	0.673
	F4d	0.638
Math Score	Mt 1	0.925
	Mt 2	0.932
	Mt 3	0.937
	Mt 4	0.933

Validity

After achieved the unidimensionality for the model, the validity of the model also should be tested. Validity is a test for the construct to achieve all the minimum require in fitness indexes as mention in Table 1. Fitness indexes is one of the primary goal in confirmatory factor analysis. The results of fitness indexes for the each of the measurement model is shows in Table 4.

TABLE 4: The Assessment of Fitness for the Measurement Model after Model Modification

Name of category	Name of index	Index value	Note
Absolute fit	RMSEA	0.40	The required level is achieved
	GFI	0.970	The required level is achieved
Incremental fit	AGFI	0.960	The required level is achieved
	CFI	0.974	The required level is achieved
	TLI	0.969	The required level is achieved
	NFI	0.968	The required level is achieved
Parsimonious fit	Chisq/df	4.943	The required level is achieved

From this table revealed that the results for the fitness index for the measurement model are fulfill for all index categories. In other word, construct validity is achieved.

Convergent Validity

Convergent validity is achieved if the value of Average Variance Extracted (AVE) get the minimum level in each construct for the model. Value of AVE, can be calculate using the formula in Table 2.

TABLE 5: Average Variance Construct For Every Construct.

Construct	AVE
Motivation	0.482
Teacher's Role	0.467
Attitude	0.473
Self-Confidence	0.405
Math Achievement	0.868

Table 5 show the value of AVE for every construct. AVE should be higher than 0.5. However, the value of 0.4 is acceptable due to condition that if AVE value is less than 0.5, but composite reliability is higher than 0.6, the convergent validity of the construct is acceptable (Fornell and David, 1981). Convergent validity is achieved for every construct.

Discriminant Validity

The items in the construct should not be redundant from each other in the model. To fulfill that condition, the discriminant validity test is used. To see whether the items is redundant or not, the value of the correlation between the construct should not more than 0.85. Table 6 shows the correlation value for each of the construct.

TABLE 6: Square Root Average Variance Extracted

Construct	Motivation	Teacher's Role	Attitude	Self-Confidence	Math Achievement
Motivation	**0.694**				
Teacher's Role	0.548	**0.683**			
Attitude	0.155	0.262	**0.687**		
Self-Confidence	-0.413	-0.212	-0.018	**0.636**	
Math Achievement	-0.189	0.040	0.113	0.321	**0.932**

The bold value in the Table 6 is the square root of AVE and the other values is the correlation value between the construct. Discriminant validity is achieve if the bold value is higher than the other value in its row and column.

Reliability

Reliability also must be achieve in every measurement model. It could be achieve based on several criteria. It is achieve when it get the value of Cronbach Alpha at least 0.7. Besides that, the value of composite reliability also must above 0.6. These two value should get the minimum required values in order to achieve the reliability for a construct. The formula to calculate the composite reliability is shown in Table 2. Table 7 shows all the results for testing the reliability for the measurement models.

TABLE 7: Result of Cronbach Alpha, Composite Reability and Average Variance Extracted.

Construct	Cronbach Alpha (above 0.70)	CR (above 0.6)	AVE (above 0.5)
Motivation	0.831	0.8222	0.482
Teacher's Role	0.771	0.7757	0.467
Attitude	0.704	0.7223	0.473
Self-Confidence	0.743	0.7306	0.405
Math Achievement	0.976	0.9634	0.8682

Table 7 show all the results in models for testing the reliability achieved the minimum requirement. This show reliability for all the construct was achieved.

After the fitness indexes has been achieved, normality assessment for the data is examined before proceed to structural model. Table 8 present the normality for every items involved in measurement model. The value of skewness in the table shows the value for normality for each items. The values should be not more than 1.0 to indicate it is normally distributed. This table also show the value of multivariate kurtosis. The value shows the multivariate normality for the data. This value should be less than 50 to shows that it is normally distributed. Thus, from Table 8 revealed that all items and the dataset are normally distributed due to the values of all skewness and multivariate kurtosis less than 1.0 and less than 50, respectively.

TABLE 8: The Assessment of Normality Distribution for Items in the Respective Construct

Variable	min	max	skew	c.r.	kurtosis	c.r.
math_score4	1.000	5.000	.453	9.254	-.753	-7.680
math_score3	1.000	5.000	.463	9.454	-.715	-7.297
math_score2	1.000	5.000	.454	9.263	-.742	-7.574
math_score1	1.000	5.000	.447	9.127	-.746	-7.611
F6a	1.000	4.000	.287	5.861	-.616	-6.292
F6b	1.000	4.000	.247	5.038	-.956	-9.759
F6c	1.000	4.000	.224	4.582	-.899	-9.175
F6d	1.000	4.000	.149	3.046	-.896	-9.141
F5a	1.000	4.000	-.286	-5.844	-.991	-10.111
F5b	1.000	4.000	.186	3.789	-1.246	-12.712
F5c	1.000	4.000	-.220	-4.495	-.994	-10.146
F4a	1.000	4.000	.867	17.703	.183	1.870
F4b	1.000	4.000	.486	9.926	-.502	-5.123
F4c	1.000	4.000	.847	17.284	.706	7.208
F4e	1.000	4.000	.658	13.429	-.083	-.844
F1a	1.000	4.000	-.076	-1.552	-.675	-6.894
F1b	1.000	4.000	.193	3.932	-.424	-4.324
F1c	1.000	4.000	.241	4.914	-.553	-5.644
F1d	1.000	4.000	-.221	-4.516	-.741	-7.566
F1e	1.000	4.000	-.159	-3.241	-.871	-8.888
Multivariate					41.182	34.706

Confirmatory factor analysis model

The test of unidimensionality, validity and reliability is use to get the final model. The final model is fit with the data. All the items after run the CFA give the effect to the factors and the factors is not redundant to each other. The final model of CFA is shows in Figure 1.

Figure 1 shows the confirmatory factor analysis model after modification to get the model fit. Table 9 is index for goodness of fit of the model. The correlation between the motivation, teacher's role, attitude and self-confidence are shown in Table 10. The results show that the correlation between all the exogenous construct are moderate and weak.

Based on that results, its shows that the data fit with the hypothesized models. All the value of fit indices achieved. Motivation, teacher's role, attitude and self-confidence is contributed to the student's achievement in mathematics. Based on the value of factor loading, the item in attitude construct shows the highest value which is 0.804 (Table 3). This indicates that it's give the highest impact compare to other items. It's revealed that the attitude play very important role to determine the student achievement. Attitude only can control by themselves.

Besides that, motivation, teacher's role and self-confidence also important. However, confirmatory factor analysis did not show beta estimate and significant results of each of the factors. Confirmatory can be extend to structural equation modelling analysis to get these results.
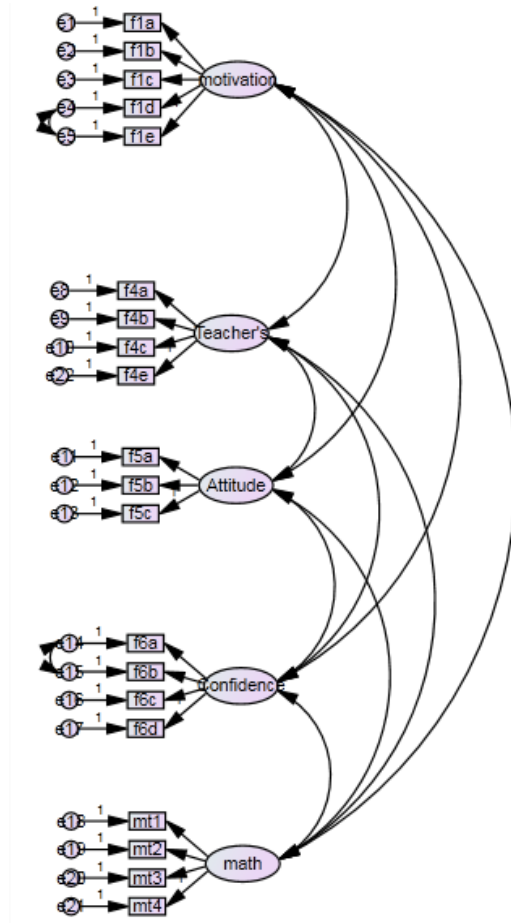

Figure 1: Confirmatory Factor Analysis Model for Students Achievement in Mathematics

TABLE 9: The Fitness of indexes for model

Name of category	Name of index	Index value	Note
Absolute fit	RMSEA	0.40	The required level is achieved
	GFI	0.970	The required level is achieved
Incremental fit	AGFI	0.960	The required level is achieved
	CFI	0.974	The required level is achieved
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Parsimonious fit	Chisq/df	4.943	The required level is achieved

TABLE 10: The Correlation Estimate For Each Pair Of Exogenous Construct

			Estimate
moti	<-->	Teachers	.548
teachers	<-->	Attitude	.262
attitude	<-->	self confidence	-.018
moti	<-->	Attitude	.155
moti	<-->	self confidence	-.413
teachers	<-->	self confidence	-.212

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